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MODELING NONPERTURBATIVE QCD FOR MESONS AND COUPLINGS

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We discuss aspects of a covariant QCD modeling of meson physics by illustrating applications to several coupling constants and form factors. In particular, we cover the $\rho\pi\pi$ and $\pi^0\gamma\gamma$ interactions, the ρ contribution to the pion charge radius, and πNN coupling.

1 Introduction

The Dyson-Schwinger equation (DSE) approach[1] to non-perturbative QCD modeling provides semi-phenomenological gluon and quark two-point functions that have proved to be quite efficient in describing and predicting the physics of low-mass mesons[2]. To outline the basis of such efforts, consider the fully-dressed and renormalized quark propagator defined by the Dyson-Schwinger equation (DSE) in Euclidean metric

$$S^{-1}(p) = Z_2[i\gamma\cdot p + m_0(\Lambda)] + Z_1\,\frac{4}{3}\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\,g^2 D_{\mu\nu}(p-k)\gamma_\mu S(k)\Gamma^g_\nu(k,p), \quad (1)$$

where $m_0(\Lambda)$ is the bare mass parameter and Λ characterizes the regularization mass scale. Here the dressed gluon propagator $D_{\mu\nu}(q)$ and the dressed quark-gluon vertex $\Gamma^g_\mu(k,p)$ are the renormalized quantities and they satisfy their own DSEs which require at least some higher order n-point functions. The ultraviolet behavior of $D_{\mu\nu}(q)$ and $\Gamma^g_\mu(k,p)$ can be constrained by perturbation theory. However physics at the hadron length scale depends crucially upon the behavior of $S(p)$, $D_{\mu\nu}(q)$ and $\Gamma^g_\mu(k,p)$ in the infrared. Present QCD modeling begins with an explicit solution of the quark DSE via phenomenological IR forms for $D_{\mu\nu}(q)$ and $\Gamma^g_\mu(k,p)$. Although the DSE itself can probe the pseudoscalar component of the pion via the chiral condensate and dynamical chiral symmetry breaking, the Bethe-Salpeter equation (BSE) is required to address other mesons and to obtain the sub-leading pion components.

The BSE for a bound state of a quark of flavor f_1 and an antiquark of

flavor $\bar{f}_2$ is

$$\Gamma(p;P) = \int \frac{d^4q}{(2\pi)^4} K(p,q;P) S_{f_1}(q+\xi P)\Gamma(q;P)S_{f_2}(q-\bar{\xi}P) \ , \tag{2}$$

where $\xi + \bar{\xi} = 1$ describes momentum sharing. The kernel K operates in the direct product space of color, flavor and Dirac spin for the quark and antiquark and is the renormalized, amputated $\bar{q}q$ scattering kernel that is irreducible with respect to a pair of $\bar{q}q$ lines. At the present stage of QCD modeling, the BSE is employed in ladder approximation with bare vertices, that is

$$K(p,q;P) = -g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2}\gamma_\mu \otimes \frac{\lambda^a}{2}\gamma_\nu \ . \tag{3}$$

The treatment of the quark DSE that is dynamically matched to this is the bare vertex or rainbow approximation for then the axial vector Ward-Takahashi identity is preserved and the Goldstone theorem is manifest. The various models of this type generally use the Ansatz

$$g^2 D_{\mu\nu}(q) \to \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \frac{4\pi\alpha_{\rm eff}(q^2)}{q^2} \ , \tag{4}$$

where $\alpha_{\rm eff}(q^2)$ implements the pQCD running coupling in the UV and a phenomenological enhancement in the IR. The pion and kaon are well described in a recent work of this type. [3]

2 QCD Modeling of Mesons and Interactions

In Euclidean metric, the mass-shell condition for meson couplings requires the quark propagators in loop calculations be evaluated at complex quark momentum. To facilitate a broad survey of such applications, the present approach is to make use of a convenient analytic parameterization of confined solutions of the quark DSE. The broad features are taken from the solution to a simple DSE model [4] that is extremely infrared dominant, produces a propagator with no mass-shell pole, and includes gluon-quark vertex dressing determined by the Ward identity. The resulting propagator is an entire function in the complex p^2-plane describing absolutely confined [5] dressed quarks in the presence of both explicit and dynamical breaking of chiral symmetry. The finer details of more realistic DSE solutions are accommodated by typically five parameters that are are used to restore a good description of pion and kaon observables: $f_{\pi/K}$; $m_{\pi/K}$; $\langle\bar{q}q\rangle$; r_π; the π-π scattering lengths; and the electromagnetic pion form factor. [6,7]

The general form of the pion Bethe-Salpeter (BS) amplitude is

$$\Gamma_\pi^j(k;P) = \tau^j \gamma_5 \Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \Big] \,, \tag{5}$$

and the first three terms are significant in realistic model solutions [3] and are necessary to satisfy the axial Ward-Takahashi identity.[8] The latter identity, to lowest order in P at the chiral limit, yields $E_\pi(k;P=0) = B(k^2)/f_\pi$ where B is the scalar part of the quark dynamical self-energy.[8] It has been quite common to assume that only this term of Γ_π is important for pion coupling. This has been questioned by recent studies of interactions, such as that shown in Fig. 1, where we employ approximate π BS amplitudes such as those obtained from a rank-2 separable ansatz [9] for the ladder/rainbow kernel of the DSE and BSE. They preserve Goldstone's theorem and should be adequate for infrared integrated quantities. Parameters are fit to $m_{\pi/K}$ and $f_{\pi/K}$. The resulting π BS amplitude is

$$\Gamma_\pi(k,Q) = i\gamma_5 f(k^2)\,\lambda_1^\pi \; - \gamma_5\,\gamma\cdot Q f(k^2)\,\lambda_2^\pi \;. \tag{6}$$

The transverse amplitude for the ρ from the same study [9] is

$$\Gamma_\nu^\rho(k;Q) = k_\nu^T g(k^2)\lambda_1^\rho + i\gamma_\nu^T f(k^2)\lambda_2^\rho + i\gamma_5\epsilon_{\mu\nu\lambda\rho}\gamma_\mu k_\lambda Q_\rho g(k^2)\lambda_3^\rho. \tag{7}$$

The BS amplitudes are normalized in the canonical way.

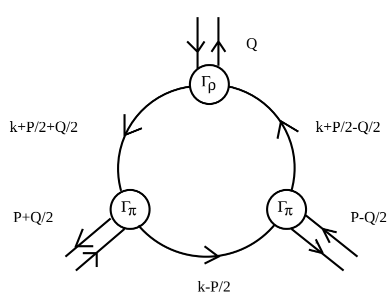

Figure 1: Diagram for the $\rho\pi\pi$ calculation.

3 The $\rho\pi\pi$, $\pi^0\gamma\gamma$ and $\gamma\pi\pi$ Interactions

The first term in a skeleton graph expansion of the $\rho\pi\pi$ vertex [10] can be expressed as

$$\Lambda_\mu(P,Q) = 2N_c \mathrm{tr}_s \int \frac{d^4k}{(2\pi)^4} S(q')\Gamma^\rho_\mu S(q'')\Gamma_\pi S(q''')\Gamma_\pi \ , \tag{8}$$

The momentum notation can easily be deduced from the Feynman rules associated with Fig. 1, e.g. $q' = k + P/2 + Q/2$ and Γ^ρ_μ represents $\Gamma^\rho_\mu(k + P/2; Q)$ etc. With both pions on the mass-shell, $P \cdot Q = 0$ and $P^2 = -m_\pi^2 - Q^2/4$. In this case symmetries require the form $\Lambda_\mu(P,Q) = -P_\mu F_{\rho\pi\pi}(Q^2)$ and the coupling constant is $g_{\rho\pi\pi} = F_{\rho\pi\pi}(Q^2 = -m_\rho^2)$.

Previous investigations of the $\rho\pi\pi$ coupling constant in terms of a covariant quark-gluon phenomenology for the intrinsic properties of ρ and π employed only γ_μ and γ_5 covariants for the respective BS amplitudes. [10,11] It has since been demonstrated for a number of infra-red sensitive quantities such as m_π and f_π, that the pseudovector terms in the pion BS amplitude are responsible for corrections in the 20-30% range. [3,8,9] The model ρ amplitude in Eq. (7) is admittedly crude, but the relative magnitude of the three surviving scalar amplitudes will hopefuly provide qualitative guidance. With the separable

Table 1: $g_{\rho\pi\pi}$ calculation and contributions from meson covariants.

$g_{\rho\pi\pi} = 6.28$ [expt 6.05]			
π Covariants		ρ Covariants	
γ_5	171%	γ_μ	94.5%
$\gamma_5\gamma\cdot Q$	-71%	$\gamma_5\epsilon_\mu\ \gamma k Q$	5.5%
		k_μ	0.01%

model BS amplitudes of Eqs. (6) and (7), the prediction for $g_{\rho\pi\pi}$, given in Table 1, compares favorably with the empirical value associated with the $\rho \to \pi\pi$ decay width. Truncation to the dominant ρ amplitude is found to only make a 5% error. However the sub-dominant pion component (pseudovector $\gamma_5\gamma\cdot Q$) enters quadratically here and makes a major contribution (-71%).

Studies of pion loops in the $\rho - \omega$ sector [10,11] and in the pion charge form factor [12] suggest that the $\bar{q}q$ extended structure of the pion significantly weakens such contributions compared to models or effective field theories built on point coupling. A similar issue arises in the role of the ρ in the space-like pion charge form factor. The dressed photon-quark vertex $\Gamma_\nu(q;Q)$ can be separated (non-uniquely) into a ρ pole or resonant piece (which is transverse) and a background or non-resonant piece (which is both longitudinal and transverse). Thus in the

present approach the pion charge form factor takes the form

$$F_\pi(Q^2) = F_\pi^{GIA}(Q^2) + \frac{F_{\rho\pi\pi}(Q^2)\,\Pi_T^{\rho\gamma}(Q^2)}{Q^2 + m_\rho^2(Q^2)}\ , \tag{9}$$

where $\Pi_T^{\rho\gamma}(Q^2)$ is the $\rho\gamma$ polarization tensor and $F_\pi^{GIA}(Q^2)$ is the generalized impulse approximation (GIA) result due to the non-resonant photon-quark coupling. It has been found to be phenomenologically successful in the space-like region and a persistent result is that $85 - 90\%$ of the charge radius is naturally explained that way.[6]

Is the ρ contribution small enough in the present QCD-modeling approach? With $F_{\rho\pi\pi}(Q^2) = g_{\rho\pi\pi} f_{\rho\pi\pi}(Q^2)$ and $\Pi_T^{\rho\gamma}(Q^2) = -Q^2 f_{\rho\gamma}(Q^2)/g_V$, which is consistent with electromagnetic gauge invariance, both form factors f depend on meson substructure dynamics and have been calculated. The ρ contribution to r_π from Eq. (9) is then

$$(r_\pi^{pole})^2 = r_\pi^2 - (r_\pi^{GIA})^2 = 1.2\ f_{\rho\pi\pi}(0) f_{\rho\gamma}(0)\ \frac{6}{m_\rho^2}\ , \tag{10}$$

where we have used the empirical result $g_{\rho\pi\pi}/g_V \sim 1.2$ rather than universal vector coupling. Our calculations include the extended nature of the mesons and produce $f_{\rho\pi\pi}(0) \approx 0.5$ and $f_{\rho\gamma}(0) \approx 0.65$. This yields $(r_\pi^{pole})^2 = 0.16$ fm^2. In contrast, the empirical Vector Meson Dominance (VMD) picture has $r_\pi^{GIA} = 0$ and $f_{\rho\pi\pi} = f_{\rho\gamma} = 1$ so that $r_\pi^2 \sim 6 g_{\rho\pi\pi}/(m_\rho^2 g_V)$ and produces ~ 0.4 fm^2. Adding the non-resonant impulse result[6] $(r_\pi^{GIA})^2 = 0.31$ fm^2 gives a total of 0.47 fm^2 with our present approach. This is obviously an overestimate of the experimental value (0.44 fm^2) leaving no room for the pion loop contribution of the expected[12] size. However, the main point is that a ρ contribution to the pion charge radius which is much smaller than that from the simple VMD assumption is consistent with the present status of DSE-based QCD modeling of the pion.

The coupling constant for the $\pi^0 \to \gamma\gamma$ decay is given by the axial anomaly and is a consequence of gauge invariance and chiral symmetry in quantum field theory. A modeling of nonperturbative QCD should preserve these features. This was the aim of a study[13] that applied the present approach and also investigated the form factor for the transition $\gamma^*\pi^0 \to \gamma$. The calculation employed a dressed quark loop similar to Fig. 1. The dressed photon-quark vertex was represented by the Ball-Chiu Ansatz which satisfies the relevant symmetries and obeys the Ward-Takahashi identity and is completely specified by the amplitudes of the dressed quark propagator. Only the first term of Eq. (5) for Γ_π was retained and the chiral limit axial Ward identity result $E_\pi(k;P) \to B(k^2)/f_\pi$ was used.

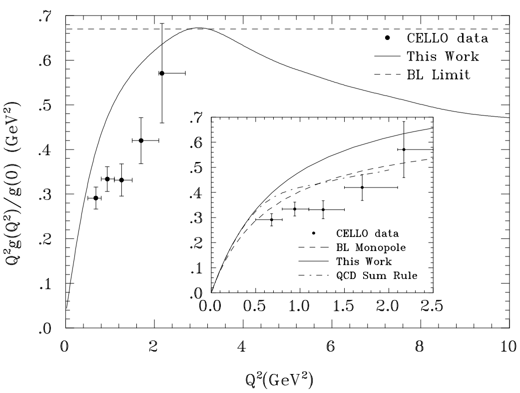

Figure 2: The $\pi^0\gamma\gamma$ transition form factor.

It was verified that the axial anomaly result for the coupling constant was obtained (independent of the details of the quark propagator parameterization used) as a test of the numerical work. The remaining pion BS amplitudes from Eq. (5) evidently do not contribute to the chiral limit coupling constant and this has been recently demonstrated analytically.[14] However such terms can become increasingly important at higher mass scales and, for example, are crucial for the asymptotic behavior of the pion charge form factor.[14] For this reason, the large momentum behavior of the calculation[13] of the axial anomaly transition form factor shown in Fig. 2 can be expected to receive significant corrections when the sub-leading pion BS amplitudes are included. This is a difficult task presently under study.

4 πNN Coupling

One can ask whether the dynamical content of the simple pion BS amplitude of Eq. (6) produces a πNN coupling constant consistent with the well-established empirical result $g_{\pi NN} = 13.4$. We make an estimate using the valence quark states of a mean field chiral quark-meson model[15,16] of the nucleon in which the chiral meson modes are generated as $\bar{q}q$ correlations. This approach has previously proved fruitful for the ρNN and ωNN couplings.[17] In Euclidean metric, the πNN vertex is

$$\vec{\Lambda}_{\pi NN}(Q) = \frac{1}{Z_N}\langle N|\int \frac{d^3p}{(2\pi)^3}\; \bar{q}(p+\frac{Q}{2})\; \vec{\Gamma}_\pi(p;Q)\; q(p-\frac{Q}{2})|N\rangle \qquad (11)$$

where q is the quark field, Q is the π momentum, $|N\rangle$ is the static mean field nucleon state and $\vec{\Gamma}_\pi$ is the BS amplitude. The nucleon valence quark wave function renormalization constant Z_N arises from the dynamical nature of the quark self-energy.[15] At the π mass-shell, Γ_π is normalized in the canonical way such that it is the residue of the pseudoscalar $\bar{q}q$ propagator there.

The standard form factor $F_{\pi NN}(Q^2)$ is identified from recasting the results from Eq. (11) into the form

$$\vec{\Lambda}_{\pi NN}(Q) = \bar{u}_N(\frac{\vec{Q}}{2})[\ i\gamma_5 \vec{\tau}_N F_{\pi NN}(Q^2)\ ]u_N(\frac{-\vec{Q}}{2})\ . \tag{12}$$

The nucleon mass shell condition does not allow distinction to be made be-

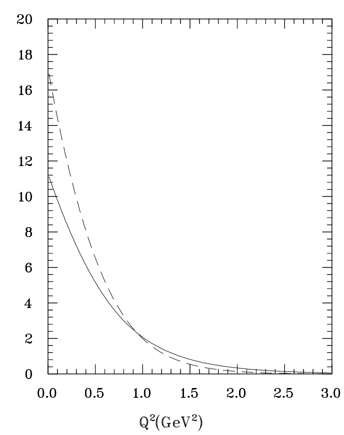

Figure 3: Pion-nucleon form factor normalized such that $g_{\pi NN}$ is the value at the pion mass shell value $Q^2 \approx 0$. The solid line includes both PS and PV components of the pion, while the dashed line includes the PS part only.

tween pseudoscalar or pseudovector coupling, and only one form factor can be identified. Such is not the case for the constituent quarks and there will be distinct contributions to $F_{\pi NN}$ from the PS and PV components of the π BS amplitude. As a mean field nucleon model cannot properly address the recoil issue, we resort to the common prescription in which Breit frame kinematics is used to define a form factor at low momentum transfer.

In Fig. 3 we show the result[18] for $F_{\pi NN}(Q^2)$ for space-like π momentum. With both PS and PV components of the pion included we obtain $g_{\pi NN} \approx 11$, while use of just the PS pion gives $g_{\pi NN} \approx 17$. Rather than compare directly to the empirical value 13.4, the only conclusion we draw from this estimate

is that, without the PV component of the π BS amplitude, $g_{\pi NN}$ would be overestimated by almost 50%.

5 Summary

Since the parameters in this approach have been previously fixed through the requirement that soft chiral quantities such as $m_{\pi/K}$, $f_{\pi/K}$ and charge radii $r_{\pi/K}$ be reproduced, the meson couplings discussed here have been produced without adjusting parameters. The results imply that this present approach to modeling QCD for low-energy hadron physics can capture the dominant infrared physics. The PV component of the pion is found to be important (at the level of about 25% and above) for a variety of physical quantities such as $m_\pi, f_\pi, g_{\rho\pi\pi}$, and $g_{\pi NN}$. We expect that the large momentum behavior of form factors such as $\gamma\pi\pi$ and $\pi\gamma\gamma$ will require attention to both types of PV amplitude evident in Eq. (5).

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